Some plans for Dynamics in HIRLAM-B

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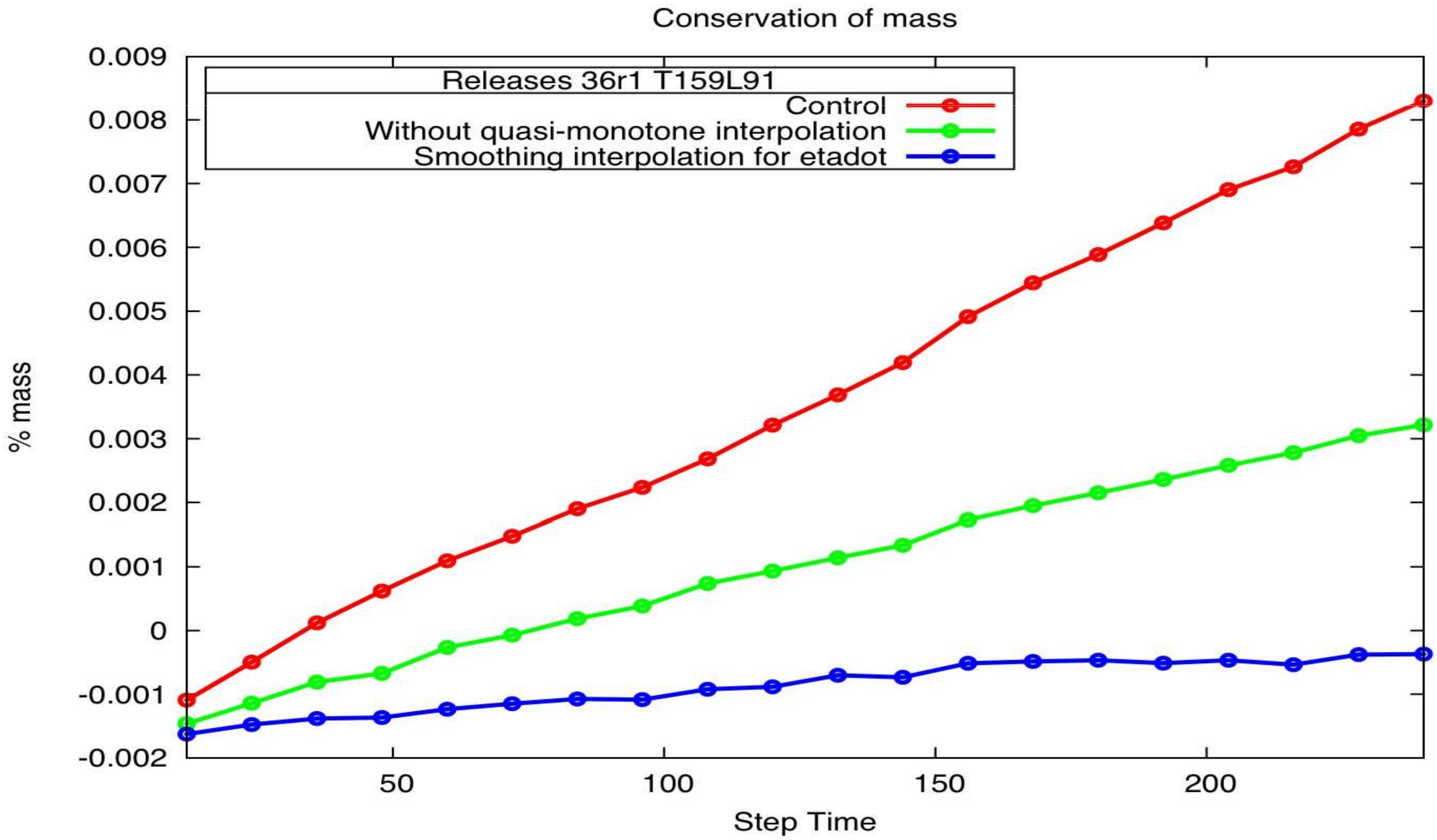
Overview

- Improvement of mass conservation in semi-Lagrangian
- Physics-dynamics interface
  - Second-order accurate treatment of slow processes
  - Physics and dynamics on different grids
- Height-based vertical coordinate with finite elements
- Semi-analytical time stepping scheme
- Laplace transform time stepping scheme

Conservation in semi-Lagrangian



Physics-dynamics interface

- Second-order accurate treatment of the physical tendencies
  - Physical tendencies (at least the slow processes) treated as the non-linear terms (SETTLS)
- Physics and dynamics run on different grids
  - Bi-linear interpolation to go from the coarser to the finer grid
  - Area averaging to go from the finer to the coarser grid

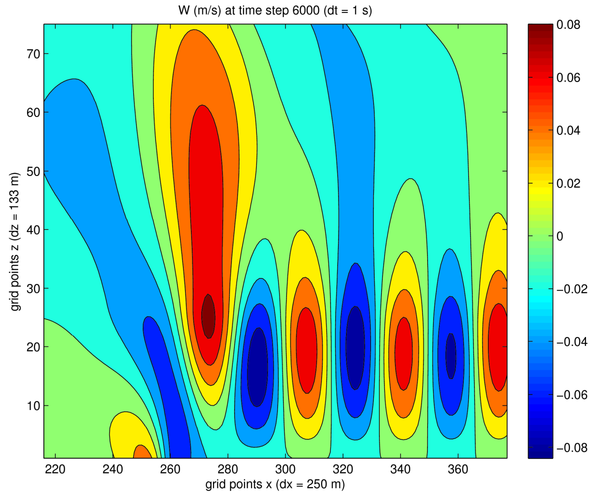
Height based coordinate and covariant variables

- Both mass and height based coordinates has been shown to be suitable for NH modeling
- Why to introduce height based coordinate in HARMONY?
  - Is a time independent coordinate system
  - Easier mathematical treatment of covariant variables
  - Covariant variables makes the expression of divergence operator simpler, without the non-linear term which is unstable in presence of orography
- Covariant vertical velocity variable W is the prognostic variable used in the Helmholtz structure equation of the semi-implicit solver
  - Robust boundary condition implementation: W=0 at boundaries is included in the semi-implicit solver.

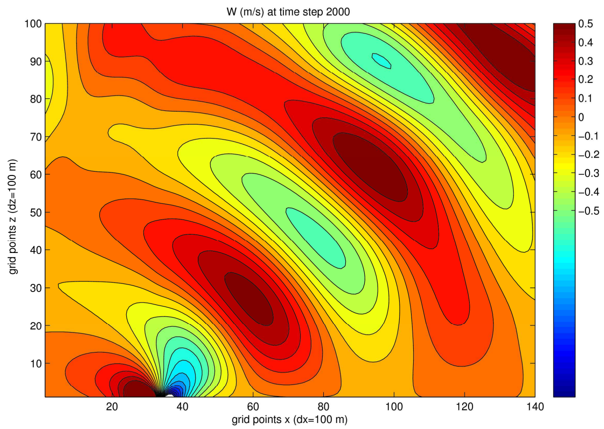
2D model for testing

- SHB numerical linear stability analysis shows:
  - amplification factors under 1.01 in most cases: stable or marginally unstable.
  - stability for temperature T in the range 0.5 T*< T < 1.5 T*.
  - In 2 time-levels stability with T*>T
- Vertical discretization:
  - VFE has been implemented, is more accurate and slightly more stable than VFD.
- Advection:
  - Covariant Eulerian and covariant semi-Lagrangian schemes has been applied successfully.

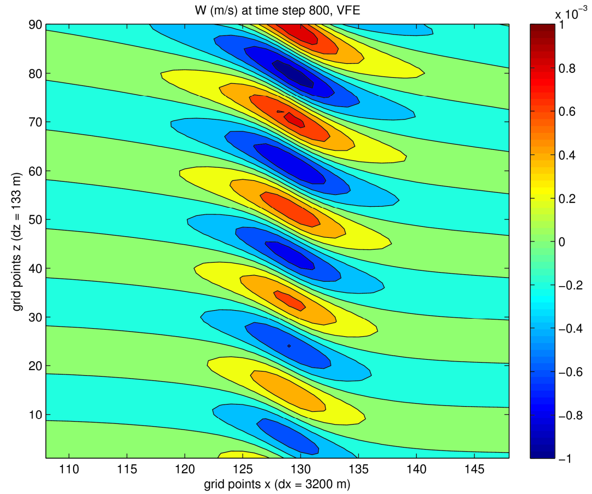
Tests done



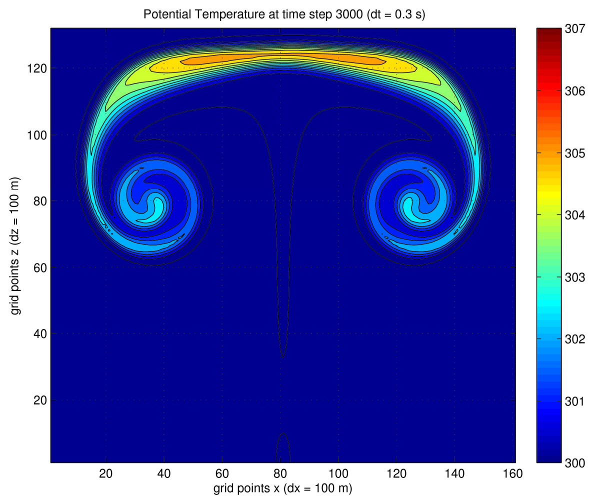
2 layer atmosphere with different stability parameters



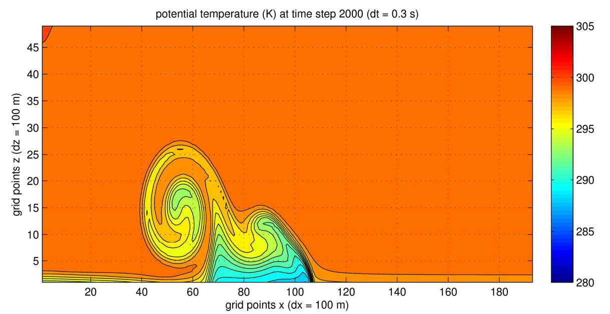
Quasi-linear non-hydrostatic flow



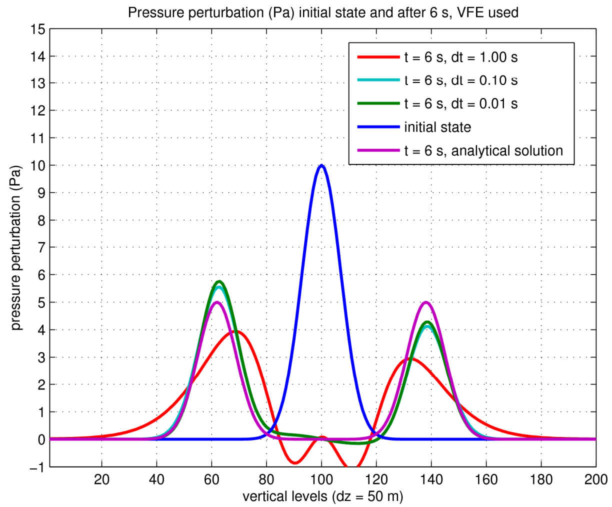
Hydrostatic flow



Warm buble



Cold buble



Acoustic wave

Semi-analytical time stepping scheme

- Once the r.h.s. of the equations has been split between linear and non-linear terms
- Taking as constant (during the time-step) the non-linear terms
  – In spectral space and projected on the eigenvectors of the vertical linear operator
- The equations are ordinary linear differential equation which have analytical solution
- No need then to apply an implicit treatment !!!!

Semi-analytical time stepping scheme (cont)

- Example: Hydrostatic IFS
  - Linearized equations ignoring advection and Coriolis

$$\frac{\partial D}{\partial t} = -\gamma\nabla^2 T - R_d T_r \nabla^2 \ln p_s + NLD$$

$$\frac{\partial T}{\partial t} = -\tau D + NLT$$

$$\frac{\partial \ln p_s}{\partial t} = -\nu D + NLP$$

Semi-analytical time stepping scheme (cont)

Semi-implicit

$$D^{+} = D^{0} - \frac{\Delta t}{2}\gamma\nabla^{2}T^{+} - \frac{\Delta t}{2}R_{d}T_{r}\nabla^{2}\ln p_{s}^{+} - \frac{\Delta t}{2}\gamma\nabla^{2}T^{0} - \frac{\Delta t}{2}R_{d}T_{r}\nabla^{2}\ln p_{s}^{0} + NLD\cdot\Delta t$$

$$T^{+} = T^{0} - \frac{\Delta t}{2}\tau D^{+} - \frac{\Delta t}{2}\tau D^{0} + NLT\cdot\Delta t$$

$$\ln p_{s}^{+} = \ln p_{s}^{0} - \frac{\Delta t}{2}\nu D^{+} - \frac{\Delta t}{2}\nu D^{0} + NLP\cdot\Delta t$$

$$\left(1 - \frac{\Delta t^{2}}{4}\nabla^{2}\left(\gamma\tau + R_{d}T_{r}\nu\right)\right)D^{+} = \left(1 + \frac{\Delta t^{2}}{4}\nabla^{2}\left(\gamma\tau + R_{d}T_{r}\nu\right)\right)D^{0} -$$
$$- \Delta t\nabla^{2}\left(\gamma T^{0} + \frac{\Delta t}{2}\gamma NLT + \Delta t R_{d}T_{r}\ln p_{s}^{0} - \frac{\Delta t}{2}R_{d}T_{r}NLP\right) + NLD\cdot\Delta t$$

Semi-analytical time stepping scheme (cont)

Semi-analytical

$$\frac{\partial^2 D}{\partial t^2} = -\nabla^2 \gamma(-\tau D + NLT) - R_d T_r \nabla^2 \left(-\nu D + NLP\right) + \frac{\partial\left(NLD\right)}{\partial t} =$$

$$= \nabla^2 \left(\gamma\tau + R_d T_r \nu\right) D - \nabla^2 \gamma NLT - R_d T_r \nabla^2 NLP + \frac{\partial\left(NLD\right)}{\partial t} =$$

$$= \nabla^2 \left(\gamma\tau + R_d T_r \nu\right) D + NL$$

Whose general solutions are

$$D(t) = Ae^{i\omega(t-t_0)} + Be^{-i\omega(t-t_0)} + \frac{NL}{\omega^2}$$

$$\text{where} \quad -\omega^2 \ \text{are the eigenvalues of} \ \nabla^2\left(\gamma\tau + R_d T_r \nu\right)$$

Semi-analytical time stepping scheme (cont)

$$At \quad t = t_0$$

$$D(t_0) = A + B + \frac{NL}{\omega^2}$$

$$i\omega A - i\omega B = -\nabla^2 \gamma T(t_0) - R_d T_r \nabla^2 \ln p_s(t_0) + NLD$$

$$D(\Delta t) = D(0)\cos(\omega\Delta t) - \Delta t\left[\nabla^2 \gamma T(0) + R_d T_r \nabla^2 \ln p_s(0) - NLD\right]\frac{\sin(\omega\Delta t)}{\omega\Delta t} -$$
$$-\left(\nabla^2 \gamma NLT + R_d T_r \nabla^2 NLP\right)\frac{1-\cos(\omega\Delta t)}{\omega^2}$$

Laplace transform time stepping scheme (from P. Lynch)

The Laplace transform of the general equation

$$\frac{d\mathbf{X}}{dt} + \mathbf{L} \cdot \mathbf{X} + N(\mathbf{X}) = 0$$

Gives, after rearranging

$$\hat{\mathbf{X}} = \left(s\mathbf{I} + \mathbf{L}\right)^{-1}\left[\mathbf{X}^0 - N^0 / s\right]$$

Applying the inverse operator at $t = \Delta t$

gives the filtered state

$$\mathbf{X}(\Delta t) = \mathbb{L}^* \left\{\hat{\mathbf{X}}\right\}\Big|_{t=\Delta t}$$